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Contents

1. Outline of Tasks in REMESH Related to Risk Based Decision Making

This report presents the outcomes of Work Package (WP) 6 of the REMESH project: Emergency decision-making based on multiple criteria.

It is deliverable D6.1: Risk-based decision-making (stage 1). It is directly linked to Objective 2: To develop a generic risk-based decision framework, and Objective 3: To develop techniques capable of dealing with a variety of multiple criteria of both qualitative and quantitative data. It also contributes to Objective 1: To strengthen the knowledge base in ERSC (Emergency Resources Supply Chain). It fulfils Task 6.1: Developing a general decision-making framework with a hierarchical structure, where major criteria can be broken down to more detailed levels, and Task 6.2: Developing techniques for modelling qualitative criteria and combining them with quantitative criteria in a rational and unified framework.

It first outlines the main objectives and tasks related to "Risk Based Decision Making", challenges in achieving those objectives and accomplish those tasks, and proposes methods and techniques to handle those challenges. Those methods and techniques can be the research focus of exchange researchers.

The *REMESH* aims to bring together an international team of researchers with a wide variety of expertise to investigate emergency resources supply chain (ERSC) and develop an ERSC management framework.

From the - project work plan stated on the grant agreement, we can see that this project has the following general and specific **objectives and tasks related to risk and uncertainty modelling and decision making**.

GO3 – To build joint collaborative projects **for the development of novel risk modelling and decision- making techniques, aimed at supporting sustainable emergency response management subjected to various aggressive environments**.

SO4 - To investigate the modelling of multiple criteria such as reliability, environmental criticality, economic impact and social sustainability of ERSC of a diverse nature, and develop risk-based decision-making models for their integrated consideration.

In the context of ERSC, hazard identification and safety assessment, cost-benefit analysis, reliability, environmental criticality, economic impact and social sustainability can all be modelled and analysed through multiple criteria decision modelling and analysis.

This project proposes and implements a decision support framework to capture and minimize inherent vulnerabilities and improve resilience in the ERSC for large-scale natural disasters. **Uncertainty modelling, expert knowledge elicitation**, cold chain management, **risk prediction, software tools, probabilistic and possibilistic risk estimation, cost-benefit modelling and multiple criteria decision making (MCDM)** will be addressed and investigated in relation to ERSC logistics risk and safety. Big data technology will be applied to understand the behaviour of ERSC.

The work developed by each of the project researchers will be **integrated** during the exchange period, in order to formulate an interactive structure, where effective risk modelling and decision making are achieved in a **dynamic framework** (see Figure 1).

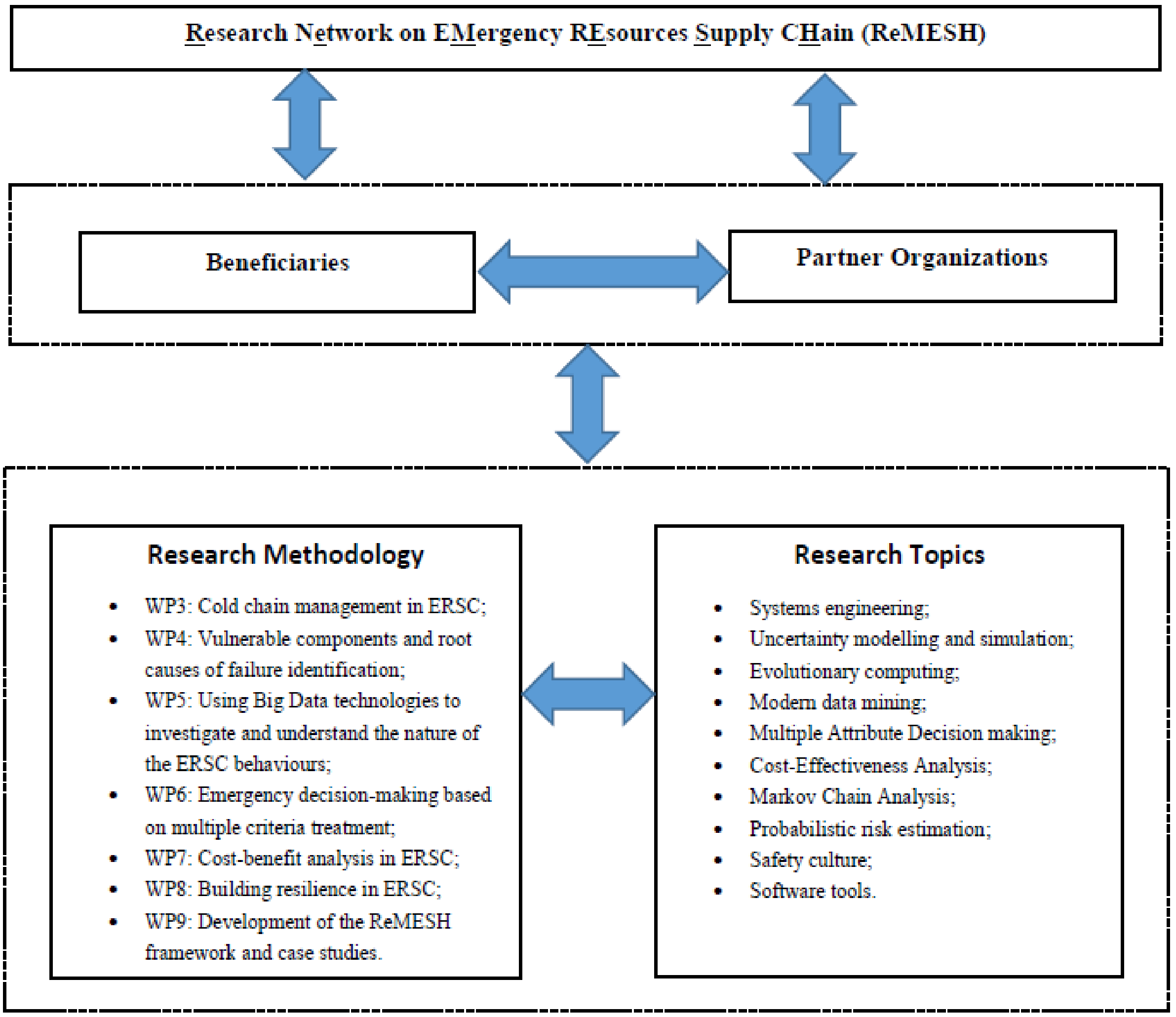

Figure 1 A research framework of the project (copied from submitted project proposal)

2. The main challenges

We envisage many challenges related to risk based decision making. For example, data generated from routine activities are hardly perfect and are almost always associated with uncertainty (Shafer 1976; Yang and Xu, 2013). It is essential to deal with different types of uncertainty consistently in system modelling and decision making. Conventional data-driven modelling tools such as multivariate regression, support vector machine and artificial neural networks model relationships among system inputs and outputs in a non-probabilistic manner, thus with limited power in handling various types of uncertainty in data, such as randomness, inaccuracy and ambiguity. Logistic regression and Bayesian networks are classical probabilistic modelling tools but are not well suited to handling inaccuracy and ambiguity, which can be caused by factors such as errors and missing values in routinely generated data.

The following are a few of those challenges.

i) One grand challenge is how to use a mix of big data and domain knowledge to develop decision making models so that a decision making process is transparent, informative and consistent, and be made rationally.

ii) Good decisions are based on good data. Obtaining **real world high quality data** could be challenging.

iii) There are various types of uncertainty associated with decisions in emergency resources supply chain, such as incomplete information, randomness of risky events, subjective, qualitative nature of judgements, and the mix of them. Identify those types of uncertainty, explicitly model them in a respectable manner and analyse their effects on decision making rationally and rigorously could be very challenging.

iv) As data are collected routinely and cheaply, large amount of data, not necessarily well-structured or from well-designed experiments, become available. How to **take into account the reliability and quality of data**, and **elicited useful and unbiased information** from the data is also challenging. It is well studied that people often make biased estimate of probabilities of risk events. With the large amount of data available, can the probability be estimated more accurately?

v) How to identify the factors that can predict the probability of a risk event from historical data? How to estimate the probability of a risk given that some phenomena are observed? Different factor may play different important roles and they may interact in causing a risk event to happen. How to identify the importance and the interaction levels

Some of the decision modelling and analysis methods may be capable of handling those challenges, but they need to be tested and validated by data or in real world applications.

3. Methodologies

We recommend to test a number of methods to verify their capability in dealing with some of the challenges outline above.

3.1 Evidential Reasoning Approach for Multiple Criteria Decision Analysis

The Evidential Reasoning (ER) approach (Yang and Xu, 2002; Xu, 2012) is a general approach for analyzing multiple criteria decision making (MCDM) problems under uncertainties. Traditionally, MCDM problems are represented or modeled by decision matrices, including pairwise comparison matrices used in AHP (Saaty, 1988; Farkas and Rózsa, 2001), in which exact numbers without uncertainties are frequently used as their elements and are incapable of explicitly modeling uncertainties like ignorance and probability distributions. The subsequent outcomes from analysis based on such models appear to be free of uncertainties, which could be misleading to the inexperienced. Even to the experienced, although further sensitivity analysis can be carried out to reveal some of the effects of uncertainties which were not modeled in the first place, the anchoring effects (Bazerman 2005) of the outcomes could be significant and lead to biased decisions. At the same time, sensitivity analysis is by far from ideal for identifying the combined effects of various types of uncertainty which often co-exist in a decision making problem.

The ER approach is developed on the basis of Dempster-Shafer evidence theory (Shafter 1976) and decision theory. By introducing the concepts of belief structure (Yang and Xu 2002; Zhang, Yang and Xu 1989) and belief decision matrix (Xu and Yang 2003), it becomes possible to model uncertainties of various types of nature in a unified format for further analysis without resorting to sensitivity analysis.

This initiative provides a new avenue for exploring how various types of uncertainty can be handled in an integrated way. Since the introduction of the modeling technique with belief structure in 1989 (Zhang, Yang and Xu 1989) and the development of the ER approach, significant amounts of work in this area have emerged in literature, including the Evidential Reasoning Rule (Yang and Xu 2013) and Belief Rule Based (BRB) expert system (Yang, Liu, Wang, Sii and Wang, 2006; Xu, Liu, Yang, Liu, Wang, Jenkinson and Ren 2007).

The ER approach should be able to contribute to the following REMESH goals and tasks: ***Uncertainty modelling, novel risk modelling, safety assessment, cost-benefit analysis, evaluation of economic impact and social sustainability of ERSC, development of risk-based decision-making models. It can also contribute to deal with challenges*** **i)** ***and*** **iii)** ***listed in the previous section.***

The key concepts of the ER approach, belief Structure and Belief Decision Matrix, and how various types of uncertainty are modelled by using a unified belief structure, the ER algorithm for decision making based multiple pieces of evidence are given in appendices A3.1.1 to A3.1.4

3.2 Belief rule based expert (BRB) system:

BRB system **(Xu, 2012)** is an expert system which can be used to model complex uncertainty and nonlinear relationship between input and output variables of a system. In recent years, it has been developed as an interpretable machine learning method for learning the relationship from data. ***For REMESH project, it should provide a method to develop risk-based decision-making models for their integrated consideration, and for risk prediction. It should have the capability to learn from data and also offer interpretability to the causes or observable signs of risks.***

Traditionally, IF-THEN rules have been frequently used to construct knowledge based systems. In 2006, Yang et al. (2006) proposed a new knowledge representation scheme by building a probabilistic IF-THEN rule, referred to as belief rule, using belief structure (see Appendix A3.1.1). In a belief rule, all possible consequents are associated with belief degrees, and the weights of both antecedent attributes and rules are also evaluated. Such a belief rule base is capable of capturing vagueness, incompleteness, and nonlinear causal relationships between antecedent attributes (or input variables) and consequents (or output variables), and traditional IF-THEN rules can be represented as a special case (Yang et al., 2006). Formally, a belief rule can be defined as follows,

$$R_k: \begin{array}{l} \text{IF } x_1 \text{ is } A_1^k \wedge x_2 \text{ is } A_2^k \wedge \cdots \wedge x_{T_k} \text{ is } A_{T_k}^k, \\ \text{THEN } \{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), \ldots, (D_N, \beta_{N,k})\}, \left(\sum_{n=1}^{N} \beta_{n,k} \le 1\right), \\ \text{with rule weight } \theta_k, \\ \text{and attribute weight } \delta_{1,k}, \delta_{2,k}, \ldots, \delta_{T_k,k}, k \in \{1, \ldots, L\}. \end{array} \quad (1)$$

where $x_1, x_2, \ldots, x_{T_k}$ denote the antecedent attributes in the kth rule. These attributes belong to the whole set of antecedent attributes $X = \{x_i; i = 1, \ldots, T\}$, in which each element takes values (or propositions) from an array of finite sets $A = \{A_1, \ldots, A_T\}$. The vector

$A_i = \{A_{i,n}; n = 1,..., N_i = |A_i|\}$ is defined as the set of referential values for antecedent attribute x_i. In the kth rule, A_i^k represents the referential value taken by the ith antecedent attribute x_i. T_k denotes the total number of antecedent attributes in the kth rule. $\beta_{n,k}$ represents the belief degree to which D_n is believed to be the consequent, given the logical relationship of the kth rule $F_k : x_1$ is $A_1^k \wedge x_2$ is $A_2^k \wedge \cdots \wedge x_{T_k}$ is $A_{T_k}^k$. If $\sum_{n=1}^{N} \beta_{n,k} = 1$, the kth rule is said to be complete; otherwise, it is incomplete. The extreme case $\sum_{n=1}^{N} \beta_{n,k} = 0$ denotes total ignorance on the consequent. Note that the element D_n in the set of consequents $D = \{D_n; n = 1,..., N\}$ can either be a conclusion or an action (Yang et al., 2006).

As defined above, a belief IF-THEN rule represents a functional mapping between antecedents (inputs) and consequents (outputs) with uncertainties, and it can provide a more informative and realistic scheme than traditional IF-THEN rules. Furthermore, the parameters, including belief degrees $\beta_{n,k}$, rule weights θ_k and attribute weights $\delta_{i,k}$ can be assigned initially by experts and subsequently trained or updated using appropriate learning algorithms if data regarding the inputs and outputs of BRB systems are available.

Once a generic belief rule base $R =< X, A, D, F >$ is established, the knowledge embedded in these belief rules can be used to perform inference for a specific input vector.

Studies show that BRB can proximate any nonlinear functions infinitely closely (Chen et al. 2013).

3.3 Evidential Reasoning Rules:

Evidential Reasoning (ER) rule (Yang and Xu, 2013) is an extension to the ER approach introduced in Section 3.1. In the ER approach, importance weights of different criteria are taken into account in multiple criteria decision analysis but **we assume that data are completely reliable**. In the ER rule, data reliability is also taken into account when different factors and information from different sources need to be combined for making a decision. This is particularly important as big data are normally collected from daily operations, social media, distributed sensors etc. and may not be as reliable as the data collected from laboratories.

In addition to what the ER approach can do, the ER rule may be able to contribute to tackle the 4th challenge, the data reliability issue, outline in Section 2. It can be used as an initial tool to undertake the ReMESH tasks in hazard identification, analysis of economic impact and social sustainability of ERSC of a diverse nature, and development of risk-based decision-making models, uncertainty modelling, risk prediction, probabilistic and possibilistic risk estimation.

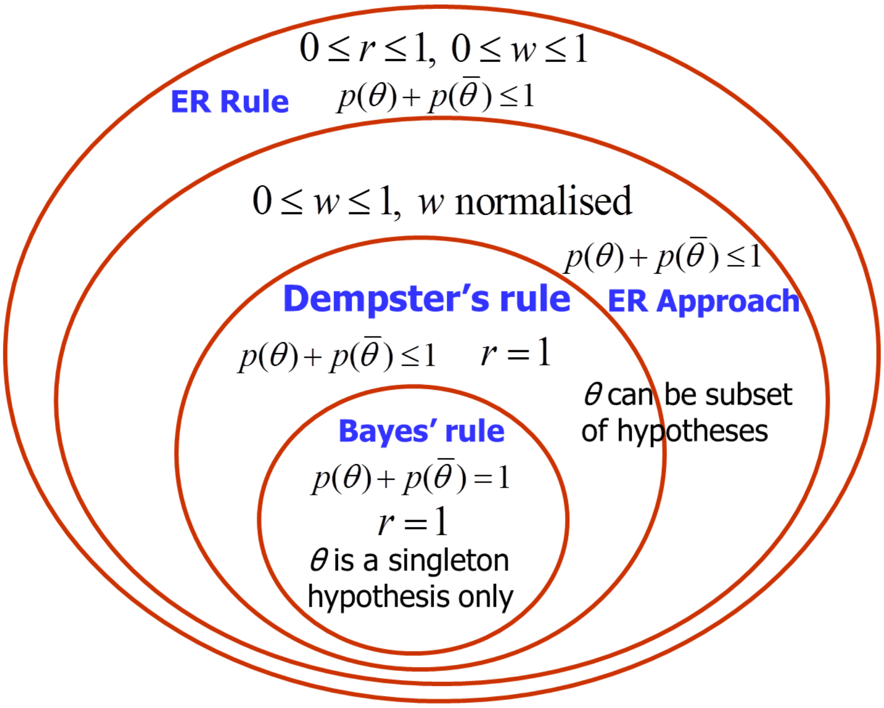

Figure 2 Relationship between ER approach, ER rule, Dempster rule and Bayes rule

3.4 Likelihood Analysis of Risk Factors and Bayes Inference for Poor Quality Data (Yang and Xu, 2014)

3.4.1 Likelihood Analysis of Risk Factors

Likelihood analysis is to decide the probability of a hypothesis being true when certain event is observed. It is based on historical data. It can identify the important risk factors. For example, suppose independent tests and diagnoses of 10000 persons in a population are shown in Table 1. We are interested to find the probability of a person having *AIDS* if the person is tested *HIV* positive.

Table 1 Experimental Data

Sample Data		Test Result		Total Diagnosis
		HIV Positive (e_1)	*HIV* Negative (e_2)	
Proposition	*AIDS* (h_1)	95	5	100
	No AIDS (h_2)	990	8910	9900
Total Test		1085	8915	10000

What needs to be identified is the probability, denoted by $p_{h_1,e(2)}$, to which h_1 is supported by both pieces of evidence: the *prior AIDS* distribution of the population as revealed by the experiment (e_0) and a positive *HIV* test result (e_1).

The *prior* probabilities, $p_{10} = p(h_1|e_0)$ and $p_{20} = p(h_2|e_0)$, of h_1 and h_2 are true respectively can be generated from the experimental data given in Table 1 as follows

$$p_{10} = p(h_1|e_0) = \frac{100}{10000} = 0.01,\ p_{20} = p(h_2|e_0) = \frac{9900}{10000} = 0.99 \qquad (2);$$

The likelihoods, c_{11} and c_{21}, of observing the outcome e_1 given that h_1 and h_2 are true respectively are calculated from the experimental data given in Table 1 as follows

$$c_{11} = p(e_1|h_1, e_0) = \frac{95}{100} = 0.95,\quad c_{21} = p(e_1|h_2, e_0) = \frac{990}{9900} = 0.1$$

If we normalize the likelihoods as follows,

$$p_{11} = \frac{c_{11}}{c_{11} + c_{21}} = \frac{0.95}{0.95 + 0.1} = \frac{0.95}{1.05} = 0.9048,\ p_{21} = \frac{c_{21}}{c_{11} + c_{21}} = \frac{0.1}{1.05} = 0.0952$$

The values of p_{11} and p_{21} inform a decision maker if an outcome, e_1 in the example, of a risk factor is observed, what is the probability of a risk (p_{11}) and not a risk (p_{21}) respectively without assuming a prior probability distribution.

If the probabilities are close to 0.5, this means that the observed outcome of the risk factor may not have much discriminating power on its own. However, if those probabilities are close to 0 or 1, this indicates that the risk factors are playing an important role in risk identification.

This example shows that likelihood analysis can partially address the 5th challenge in identifying important risk factors. It is "partially" because it does not address the problem if a number of factors jointly affect the occurrence of a risk.

3.4.2 From Likelihood Analysis to Bayes Inference

If we consider both the prior probability and the observation, the probability of a person from this population having AIDS if the person is tested HIV positive, $p_{h_1,e(2)}$, can calculated by using the ER algorithm as follows

$$p_{h_1,e(2)} = \frac{p_{11}p_{10}}{p_{11}p_{10} + p_{21}p_{20}} = \frac{0.9048 \times 0.01}{0.9048 \times 0.01 + 0.0952 \times 0.99} = 0.0876$$

From the conventional Bayesian analysis, the same result can be generated as follows

$$p(h_1|e_1, e_0) = \frac{p(e_1|h_1, e_0)p(h_1|e_0)}{p(e_1|h_1, e_0)p(h_1|e_0) + p(e_1|h_2, e_0)p(h_2|e_0)} = \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.1 \times 0.99} = 0.0876$$

3.4.3 Bayes Inference for Poor Quality Data

The following example illustrates how to conduct Bayes inference by using the ER rule when a decision has to be made based on data that may not be of high quality. ***This methods can be a starting point for handling the 5th challenges***.

Suppose there are some poor quality data for a population, as shown in Table 2. It is also assumed that the experimental data can represent the *prior AIDS* distribution of the population with a 95% level of reliability and an *AIDS* diagnosis from a *HIV* test is regarded to be 98% reliable. What is the probability of a person from the population having *AIDS* if the person is tested *HIV* positive?

Table 2 Experimental Data under Uncertainty

Diagnosis	*HIV* test result	

		Positive e_1	*Negative* e_2	*Unknown* e	Total diagnosis
AIDS	h_1	95	5	0	100
No AIDS	h_2	980	8860	10	9850
Unknown	$\Theta=\{h_1,h_2\}$	5	7	38	50
Total test		1080	8872	48	10000

In Table 2, the probabilities (or belief degrees to be more precise because of the unknown or missing data in the records) in the *prior AIDS* distribution (e_0) for the population are given by $p_{10}=100/10000=0.01$, $p_{20}=0.985$, $p_{\Theta 0}=0.005$ which are the probability of a person having aids, not having aids and unknown respectively.

Similar to the example in section 3.4.1, the likelihood $c_{\theta 1}$ or the probability of a person having positive *HIV* test given this person's real state is θ which could be h_1, h_2 or Θ (unknown), and belief degree $p_{\theta 1}$ (the probability of $\theta \in$ (h_1, h_2 or unknown) being true) for the evidence of positive *HIV* test result (e_1) are calculated in Table 2 by $c_{11}=95/100=0.95$, $c_{21}=980/9850=0.0995$, $c_{\Theta 1}=5/50=0.1$, and then

$$p_{11}=\frac{c_{11}}{c_{11}+c_{21}+c_{\Theta 1}}=\frac{0.95}{1.1495}=0.8264,\ p_{21}=0.0866,\ p_{\Theta 1}==0.087.$$

The reliabilities and weights of e_0 and e_1 are given by r_0 =0.95, r_1 =0.98 and suppose that both pieces of evidence are equally weighted, i.e. $w_0=w_1=0.5$. Note that the weights are normalized with $w_0+w_1=1$ for illustration purpose. In general, this is not always required. The degrees of individual support for θ from e_0 and e_1 are calculated by

$$m_{h_1,0}=w_0p_{10}=0.5\times 0.01=0.005,\ m_{h_2,0}=w_0p_{20}=0.4925,\ m_{\Theta 0}=w_0p_{\Theta 0}=0.0025;$$
$$m_{h_1,1}=w_1p_{11}=0.5\times 0.826=0.4132,\ m_{h_2,1}=w_1p_{21}=0.0433,\ m_{\Theta 1}=w_1p_{\Theta 1}=0.0435$$

To combine e_0 and e_1 to count their joint support, the calculations are given by

$$\hat{m}_{h_1,e(2)}=(1-r_1)m_{h_1,0}+(1-r_0)m_{h_1,1}+m_{h_1,0}m_{h_1,1}+m_{h_1,0}m_{\Theta 1}+m_{\Theta 0}m_{h_1,1}=0.0241$$
$$\hat{m}_{h_2,e(2)}=(1-r_1)m_{h_2,0}+(1-r_0)m_{h_2,1}+m_{h_2,0}m_{h_2,1}+m_{h_2,0}m_{\Theta 1}+m_{\Theta 0}m_{h_2,1}=0.0549$$
$$\hat{m}_{\Theta,e(2)}=(1-r_1)m_{\Theta 0}+(1-r_0)m_{\Theta 1}+m_{\Theta 0}m_{\Theta 1}=0.0023$$

The belief degrees to which e_0 and e_1 both support θ are finally generated by (Yang and Xu, 2014)

$$p_{\theta_1,e(2)}=\frac{\hat{m}_{h_1,e(2)}}{\hat{m}_{h_1,e(2)}+\hat{m}_{h_2,e(2)}+\hat{m}_{\Theta,e(2)}}=\frac{0.0241}{0.0241+0.0549+0.0023}=\frac{0.0241}{0.0813}=0.2964$$

$$p_{h_2,e(2)}=\frac{0.0549}{0.0813}=0.6753,\ \text{and}\ p_{\Theta,e(2)}=\frac{0.0023}{0.0813}=0.0283$$

The ambiguity and inaccuracy in the experiment are retained by $p_{\Theta,e(2)}$ in the above final results. As such, the probability to which the person has *AIDS* is not precise but between 0.2964 and 0.3247 ($p_{h_1,e(2)}+p_{\Theta,e(2)}$). The probability to which the person does not have *AIDS* is between 0.6753 and 0.7036 ($p_{h_2,e(2)}+p_{\Theta,e(2)}$).

It should be noted that the reliability of evidence plays an important part in inference and should be estimated with care and rigor. For instance, if both pieces of evidence are assumed to be fully reliable in this example, or $r_0 = r_1$ =1, it can be shown that there will be $p_{h_1,e(2)} = 0.0716$, $p_{h_2,e(2)}$ =0.926 and $p_{\Theta,e(2)} = 0.0024$, meaning a much smaller probability (0.0716 to 0.0740) of having *AIDS* with much smaller ambiguity (0.0024). Such results are quite different from the results generated above for r_0 =0.95 and r_1 =0.98, but justifiable as evidence e_0 is against the first proposition "*AIDS*" much more than evidence e_1 against the second proposition "*No AIDS*".

4. Summary and Discussion

To summarize, the key findings presented in the report is that the Evidential Reasoning approach for multiple criteria decision analysis is identified as a technique which can match the challenges of modelling qualitative criteria and combining them with quantitative criteria. The Intelligent Decision System software package is identified as a suitable tool for supporting multiple criteria decision analysis. However, the approach and the tool need to be implemented for particular decision making applications in ERSC, and decision options, decision criteria and criteria weights of the decision making problems need to be identified. Problems in consideration include pharmaceutical product procurement and emergency decision-making based on multiple criteria in railway operation in Bangkok Thailand.

Once the implementation is complete, the tool becomes fully developed for this project and operational for supporting decision making in those two areas initially: pharmaceutical product procurement and emergency decision-making based on multiple criteria in railway operation. They will act as examples and can be tailor made to support other emergency supply chain decision making. The implementation process was planned to be completed by M12 (30 April 2020) and reported in version 2 of this report. Due to Covid-19 pandemic, the implementation which requires the collaboration between UNIMAN and MU in Thailand has been postponed due to travel restrictions.
The outcomes have been presented in a seminar in Mahidol University in January 2020 and in the online REMESH project workshop "REMESH Framework and Its Implementation" in September 2020. The likelihood analysis part of the outcomes (Section 3.4) of this report is included in a paper (Yang and Xu 2020) to be submitted to IEEE Transaction on Systems, Man and Cybernetics.

The outcomes contributes to WP2: Travel integration and transfer of knowledge by presenting at a seminar and a workshop), WP5: Using Big Data technologies to conduct risk analysis by providing tools for undertaking big data analysis with particular reference to hazard and threat identification), and WP7: Cost benefit analysis in ERSC by providing tools for undertaking uncertainty modelling in cost estimation.
The researcher exchange activities related to risk based decision making will be focusing on utilizing the strengths of the project partners in their respective fields to address the challenges in risk based decision making in their fields. The application case studies will be presented in a dedicated workshop on risk-based decision-making, originally planned to take place in Month 18 (October 2020). Due to travel restrictions to contain the Covid 19 pandemic, the researcher exchange activities, the production of case studies and the organisation of the workshop have all been put on hold until the restrictions are relaxed.

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Appendix for Section 3

A3.1.1 Concept of Belief Structure

A belief structure is a **distributed assessment** using belief degrees. It is used to represent the performance of an alternative assessed against a criterion. To illustrate the concept, suppose there is a MCDM problem in which M cars (alternatives) are evaluated against L criteria and one of which is *Engine Quality*.

The engine quality of the m^{th} ($m = 1,\dots,M$) car may be assessed to be "*Excellent*" to some degree (e.g. 0.6) due to its low fuel consumption, low vibration and high responsiveness. At the same time, the quality may also be assessed to be only "*Poor*" to some degree (e.g. 0.4 or less) because its quietness and starting quality can still be improved. Such an assessment can be modeled as:

$$a_{ml} = \{(Excellent, 0.6), (Poor, 0.4)\} \qquad \text{(A3.1)}$$

where $\{(Excellent, 0.6), (Poor, 0.4)\}$ is referred to as a belief structure in which "*Excellent*" and "*Poor*" are assessment grades, whilst "0.6" and "0.4" are degrees of belief.

More generally, suppose a MCDM problem has M alternatives assessed on L criteria. Let

$$H = \{H_1, \cdots, H_N\} \qquad \text{(A3.2)}$$

be a collectively exhaustive and mutually exclusive set of assessment grades where N is the number of grades in the set. Then a belief structure can be expressed as

$$a_{ml} = \{(H_1, \beta_{l,1}),\dots,(H_N, \beta_{l,N}),(H_{1N}, \beta_{l,1N})\} \qquad \text{(A3.3)}$$

where $m = 1,\dots,M$, and $\beta_{l,i} \geq 0$ ($i = 1,\dots, N$; $l = 1, \dots, L$) is a belief degree to which the performance of the alternative is assessed to the grade H_i on criterion l, $\beta_{l,1N} = 1 - \sum_{i=}^{N} \beta_{l,i} \geq 0$ and H_{1N} is the set of grades from H_1 to H_N. H_{1N} is used to represent unknown in the assessment. When $\sum_{i=1}^{N} \beta_{l,i} = 1$ or $\beta_{l,1N} = 0$, the assessment is said to be **complete**; otherwise it is **incomplete**.

A3.1.2 Uncertainty Represented by Belief Structure

Using a belief structure, whether the performance of an alternative on a criterion is measured by precise data or data with uncertainties, it can be modelled as follows.

- **Precise data**

  If the performance can be precisely assessed to a grade, such as "Excellent", without any doubt, then it can be represented by the belief structure $a_{ml} = \{(1.0, Excellent)\}$. Therefore precise data (including qualitative data such as an assessment grade) can be seen as a special case of a belief structure. This will lead to a conclusion later in this sub section that decision matrix is a special case of belief decision matrix.

- **Absence of data.**

Absence of data is used to describe a situation where there is no data available to assess the performance of an alternative against a criterion. Such a case can be represented by a belief structure in which the sum of total **belief degrees** is 0, i.e. $\sum_{i=1}^{N} \beta_{l,i} = 0$ or $\beta_{l,1N} = 1$,

- **Partial data or incomplete data**

  This is a situation where data for measuring the performance of an alternative against a criterion are partially available. If this is the case, the sum of total **belief degrees** in the **distributed assessment** for that attribute will be between 0% and 100%, i.e. $0 < \sum_{i=1}^{N} \beta_{l,i} < 1$.

- **Probability uncertainty**

  Some outcomes measured against a criterion may be of random nature. For example, the fuel consumption of a car in mile per gallon is not a deterministic number. Depending on road conditions, traffic conditions and seasons of a year, the figure can vary. The nature of fuel consumption can be described by a probability distribution, which is a belief structure in nature. Other common sources of uncertain data can be subjective judgements or questionnaire surveys. For example, in a customer satisfaction survey, if 20% of the customers evaluate the after sale service of a computer shop to be excellent, 30% good, 40% average and 10% with no opinions, this piece of evidence can then be represented by a belief structure as follows:

  $\{(Excellent, 0.2), (Good, 0.5), (Average, 0.3), (unknown, 0.1)\}$.

  It should be noted that a belief structure can be a continuous probability or belief distribution theoretically. Practically, such as in IDS - the software implementation of the ER approach, the continuous distribution is approximated by a discrete one of up to 20 data points.

A3.1.3 Concept of Belief Decision Matrix

In a decision matrix, if each element a_{ml} is a belief structure, then it will be called a belief decision matrix (Table A1).

Table A1 Belief Decision Matrix

Alternative	**Attribute**				
	1	**…**	***l***	**…**	***L***
1	a_{11}		a_{1l}		a_{1L}
…					
m	a_{m1}		$a_{ml} = \{(H_1, \beta_{l,1}), \ldots (H_N, \beta_{l,N}), (H_{1N}, \beta_{l,1N})\}$		a_{ml}
…					
M	a_{M1}		a_{Ml}		a_{ML}

As discussed earlier, precise data are special cases of belief structures. It can be easily seen that a decision matrix using only average numbers as its elements is a special case of belief decision matrix when all belief degrees in each belief structure is either 1 or 0 subject to the condition that the sum of belief degrees in each belief structure is 1.

Table A2 Example of Belief Decision Matrix

Car Assessment					
		Engine Quality			
	Price	**Quietness**	**Fuel Consumption**	**Vibration**	**Responsiveness**
Car 1	£8000	*Very Quiet* (100%)	35 (50%), 40 (50%)	*Heavily* (50%), *Normally* (50%)	*Good* (75%) *Excellent*(25%)
Car 2	£9000	*Quiet* (100%)	40 (33%), 45 (33%), 50 (33%)	*Lightly* (100%)……	*Good* (35%) *Excellent* (65%)
⋮	⋮	⋮	⋮	⋮	⋮
Car N	£7000	*Noisy* (100%)	45 (25%), 46 (25%), 48 (25%), 49 (25%	*Heavily* (80%), *Normally* (20%)	*Average* (15%) *Good* (70%) *Excellent* (5%) *Unknown* (10%)

A3.1.4 ER algorithm and the IDS Software

Details of the ER algorithm for information aggregation are given in the papers by Yang and Xu (2002) and Xu (2002). The ER algorithm and the ER approach is implemented in a software package called Intelligent Decision System (IDS) (Xu and Yang 2003; Xu, McCarthy and Yang 2006a). Its application can be through the IDS software.

IDS has the following unique features.

- In IDS the employment of belief structure for problem modelling allows it to accept and facilitate the collection of different types of raw information, such as quantitative information with different units and probability uncertainty, and subjective judgements with uncertainty using different sets of grades.
- Assessment information can be completely known (e.g. 100% total degree of belief in a belief structure), partially known (less than 100% total degree of belief) or completely unknown (0% total degree of belief).
- The aggregated performance of each alternative is a belief structure instead of an average score, which provides more informative conclusions and is proved useful in many decision making situations. It provides a transparent and panoramic view of the performances of the alternatives and shows the diversity or profiles of their performances so that decision makers can easily identify the strengths and weaknesses of each alternative for formation of improvement strategies.